Statistical Tolerance Analysis

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Statistical Tolerance Analysis closes the loop on SPC.

STA reduces the cost to manufacture and the risk of defects! So, what's the catch?

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Most mid-sized manufacturing companies have come to grips with Statistical Process Control (SPC) but few have closed the loop on the full opportunities afforded by a well controlled manufacturing process. While many companies have taken control of the manufacturing process, implemented SPC throughout the shop floor, determined their process capabilities (Cpk), and have amassed an amazing collection of X Bar and R charts, most have not made good use of the fruits of their labor. While these organizations have made tremendous improvements in their manufacturing operations and great strides to run within control limits, they have left significant money on the table by not taking these newly learned skills back into the product development and design processes.

A couple of years ago as Director of Engineering and Quality for a major producer of hydraulic cylinders we were faced with designing a new rod bushing for a custom cylinder. The dimensional constraints intrinsic with this design meant we had designed a rod bushing with tolerances that were going to require that the components be manufactured on a high precision turning center and then ground and polished to get the precision required. Clearly, this bushing was barely manufacturable and hardly mass producible.

The net result of using this proposed technique in the new design was a component that could be made on a middle of the road turning center and could be turned out in about three minutes per part. The cost to produce dropped by a factor of 3 and the first pass yield exceeded 99%.

So, let's understand this. The cost to manufacture goes down, the risk of defects drops dramatically, what is the catch?

<u>Traditional Tolerancing</u>

Consider the following example of a design which includes three components combined into a single assembly:

A	B	C
1.000 ±0.001	0.500 ±0.0005	2.000 ±0.002

Traditional Tolerancing, called “line to line” is far to conservative given a well controlled process.

The dimensions represented by parts A, B, and C are referred to as interacting dimensions in that they merge with other dimensions to create a final result. In this simple mechanical assembly the traditional approach to tolerancing, to building ranges of intended machine accuracy into the assembly, would define the machining specifications as 3.500 ±0.0035 giving the limits of 3.5035 and 3.4965. These extremes can be further illustrated as follows:

Part	**Maximum**	**Minimum**
A	1.001	0.999
B	0.5005	0.4995
C	2.002	1.998
Total	3.5035	3.4965

While the approach of tolerancing is mathematically correct, it is far too conservative. It is referred to as “line to line” tolerancing. It is called this because the designer can allow each component to go right up to the line before there is a missed dimension and, therefore, a failure.

Examine the manufacturing processes and commensurate costs associated with the dimensions. Let us assume for the sake of argument that the cost to manufacture part A is \$0.50. Typically, just examining the relative tolerances, part B is likely to cost \$1.00 and part C will cost approximately \$0.25.

Let us assume that 1 percent of the pieces of part A will be below tolerance limits and suppose the same for parts B and C. If a single part A is selected at random there is, on average, one chance in 100 that it will be on the low side and similarly for parts B and C. If the assemblies are made at random and if the components are manufactured independently, then the chance of all three components being out of specification is:

1 / 100 X 1 / 100 X 1 / 100 = 1 / 1,000,000   one in a million.

There is only one chance in a million that all three components will be too small resulting in a small assembly. Using this tolerance approach is overly conservative in that it does not recognize the extremely low probability that all three components will be on the low side of the specifications.

Using “line to line” tolerancing does not recognize the extremely low risk of defective parts.

Statistical Tolerance Analysis

A different approach that might be used is a statistical approach based upon the relationship of the variances of a number of independent causes and the variance of the dependent, overall result. To do this we need to use some statistical approaches and understand more than just the average dimension made by the machining process. We need to understand the value of the standard deviation of the process - σ (sigma). Sigma is the calculated description of the likelihood that a particular part will deviate from the prescribed dimension by a particular percentage. For a normally distributed process plus or minus 1σ says that roughly 68 percent of the samples made will be captured within the tolerance range. So if we say that the process made to generate 3.500" actually has a ±1σ of .010" then we are saying that 68 percent of the parts made will be between 3.495" and 3.505. Actual numbers for sigma are shown below:

Sigma Range	Percent Capture
±1σ	68.26%
±2σ	95.46%
±3σ	99.73%

If we assume that the tolerance range of our sample process above (T) is equal to plus or minus 3 standard deviations (±3σ) which will capture 99.73% of all the parts made, then the relationship between the overall tolerance and the individual tolerance ranges is shown below:

STA uses a Square Root of the Sum of the Squares (SRSS) approach to combine tolerances for a specific assembly.

$$T_{assembly} = (T^2_A + T^2_B + T^2_C)^{1/2} \qquad \text{Equation 1}$$

The squares of tolerances are added to determine the square of the tolerance for the overall result. Let us examine what can happen to the tolerances without affecting the overall tolerance of the assembly.

If the total tolerance for the assembly is ± .0035" then one solution for the individual tolerances of A, B, and C is shown below:

Component	Alternative 1	Alternative 2
A	±0.001	±0.002
B	±0.0005	±0.002
C	±0.002	±0.002

If we use the tolerances shown above for Alternative 2 and combine them using Equation 1 we can see that Alternative 2 will allow a significant reduction in the cost to produce, the risk of defects, and the overall tolerance of the assembly is ±.00346" — not much of a change from .0035" of Alternative 1.

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The relative difference in cost to produce for each of the individual approaches is shown below.

Component	Process 1 Cost	Process 2 Cost
A	$0.50	$0.25
B	$1.00	$0.25
C	$0.25	$0.25
Total	$1.75	$0.75

Given this significant reduction in the cost to produce, there is still a finite likelihood of creating a combination that will not fit together. Since the probability of defect free production for each component is 99.73%, there is a finite possibility of creating a defect for each component of .0027. This means the probability of a combination that will exceed the .0035" is shown below:

There are three rules required before Statistical Tolerance Analysis will work.

$.0027 \times .0027 \times .0027 = 1.97 \times 10^{-8}$
not quite 2 in one hundred million or about 50 times better than Alternative 1

There are three rules required before this process applies:

1. The components must be independent and the components are assembled randomly. This assumption is usually met in practice.
2. Each component dimension should be normally distributed.
3. The actual average for each component is equal to the nominal value stated in the specification. That is to say that the process should be properly centered.

Once these conditions have been met we can close the loop on this process by turning back to engineering and changing the tolerances, reducing the costs to produce and perhaps even changing the machines on which the components are made. This redesign of the product will further reduce the costs of production without loosing the productivity and yield gains made in process control.

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If you would like more information about how Arthur Andersen Business Consulting can assist your organization through our Manufacturing Operations and Strategy methodologies or our Automotive and Transportation and Equipment Industry Performance Improvement services, please contact:

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